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The cohesive zone model applied to blunt cracks

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Abstract

Many practical situations involve blunt cracks, which have a finite tip radius and no stress singularity due to the presence of a notch or self-blunting behavior. In these cases, linear elastic fracture mechanics does not apply. However, the cohesive zone model (CZM) can be used to predict the onset of fracture for blunt cracks. In this paper, the CZM is applied using finite element analysis to study the fracture of blunt cracks. Quasi-static three point bending is modelled, where fracture initiates by decohesion of a thin line of cohesive elements obeying a pre-defined traction-separation law. The effect of modelling with a bilinear traction-separation law is investigated along with the cohesive energy and cohesive stress parameters for different notch tip radii. Using the fracture load and J-integral at crack growth initiation, a non-dimensional critical energy release rate is determined as a function of a non-dimensional notch radius. A comparison is also made to the stress at a distance criteria along a line proposed by Taylor, and to the stress and energy fracture criterion proposed by Williams and Patel.

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Keywords: cohesive zone, blunt crack; fracture criterion.

1. Introduction

Linear elastic fracture mechanics (LEFM) is used in the analysis of engineering problems with a sharp crack and linear elastic material behavior. The stress concentration due to the presence of the crack causes fracture to initiate at the crack tip when the material reaches a critical state. LEFM analysis can be applied to ductile materials so long as

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the process zone ahead of the crack is sufficiently small. However, many practical situations also involve blunt cracks which have a finite tip radius and no stress singularity due to the presence of a notch or self-blunting behaviour. In these cases the modelling assumption of a sharp crack in LEFM is no longer valid.

Nomenclature

c	Critical length parameter
E'	Young's modulus for plane strain
G	Strain energy release rate
G_b	Apparent strain energy release rate
G_c	Cohesive energy
δ	Displacement
δ_c	Critical opening displacement
δ_f	Maximum opening displacement
ρ	Notch tip radius
σ	Stress
σ_c	Peak cohesive traction

Critical distance methods involve the use of a material length scale with continuum mechanics and have been used by a number of authors in the analysis of fracture from blunt cracks. This was first proposed by Neuber (1958) to predict high-cycle fatigue limits for specimens with notches by taking the mean stress over a critical distance from the notch tip. Moreover, Peterson (1959) showed that a simpler method of taking the stress at a single point could be used. Taylor (2010) refers to these methods as the theory of critical distances (TCD) and has applied them in failure prediction of brittle materials with the use of finite element analysis (FEA). Taylor describes four methods or forms that are called the Point Method (PM), Line Method (LM), Area Method (AM) and Volume Method (VM). The PM uses a critical stress at a critical distance ahead of the crack tip as a fracture criterion. Williams and Patel (2013) have proposed a criterion which requires a critical stress and $G \geq G_c$ ahead of the crack for fracture initiation.

Cohesive zones can be applied to investigate the influence of crack sharpness on fracture toughness. They do not have the crack sharpness limitation of LEFM as the fracture process is defined entirely as an interaction between two surfaces along which the fracture process acts. In this paper, an investigation is made into the parameters used to define the cohesive zone using FEA. The CZM is then compared to the stress at a distance method and the stress and energy criterion by considering the variation of non-dimensional fracture toughness with non-dimensional notch radius.

2. Modelling using the Cohesive Zone

2.1. Background

The cohesive zone model is a partially mechanistic approach to modelling the fracture process and was first proposed independently by Barenblatt (1959) and Dugdale (1960). Barenblatt developed the CZM with the aim of eliminating the stress singularity associated with LEFM using a cohesive fracture theory and Dugdale proposed a strip yield zone with the aim of estimating the plastic zone size at the crack tip. Both authors are regarded as the fathers of the cohesive zone model.

The CZM represents the fracture process as the interaction between two separating surfaces that are assumed to exist ahead of the crack tip. The constitutive behaviour of the surfaces is known as the traction-separation law, which relates the stresses and displacements between the two separating surfaces. The cohesive zone initiates when the stress at the notch tip reaches the maximum stress in the traction-separation law. Following initiation, the stress at the notch tip reduces with increased separation of the surfaces according to a pre-defined softening function. This softening function is considered a material property. Fracture initiates when the stress falls to zero at the notch tip where there is complete separation of the two surfaces. The maximum stress value is also known as the peak cohesive traction and the area under the traction-separation curve is the cohesive energy or work of fracture. The cohesive zone crack tip is the location where the stress is equal to the peak cohesive traction. This begins at the notch tip and then moves away during the fracture process.

2.2. Modelling of three point bending

A 2D quasi-static finite element model of three point bending has been produced with the fracture process region ahead of the crack tip defined by a cohesive zone. The initial crack length is 10 mm, which is half the width of the beam, and the cohesive zone is modelled using a line of elements ahead of the crack tip. This was simulated using the software product Abaqus 6.11. The notched beam was modelled with the bulk composed of continuum plane strain elements with a linear elastic constitutive response using full integration. The cohesive zone is composed of special-purpose cohesive elements that have a constitutive response defined by a traction-separation law. A bilinear traction-separation law shape was used and is shown with key parameters in Fig. 1(a). Loading occurs through displacement control and the element size is reduced nearer the crack tip. To ensure the simulation reflected a quasi-static solution, the ratio of the kinetic energy to the internal energy of the material was kept below 0.01%. The fracture load was taken as when the first cohesive element ahead of the crack tip reached maximum separation. In practice, this corresponded very closely to the maximum load in the loading history. An example of the model mesh around the notch is shown in Fig. 1(b). The fracture loads were computed for notch radii ranging from 1-300 μm.

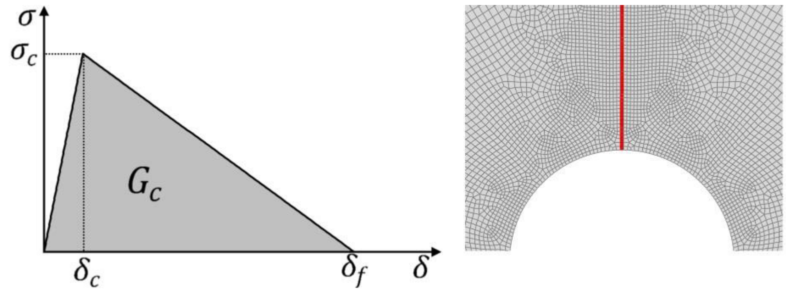

Fig. 1 (a) Bilinear traction-separation law; (b) example of mesh used in the FE model with cohesive elements highlighted ahead of the notch tip.

2.3. Effect of cohesive energy and peak cohesive traction on fracture toughness using fracture loads

An investigation was made into the effect of varying the peak cohesive traction in the traction-separation law on the apparent fracture toughness, which was calculated using the fracture loads for different notch tip radii. This was simulated using a bilinear traction-separation law while keeping the initial stiffness and cohesive energy constant. The initial elastic stiffness was kept 10^8 orders of magnitude greater than the peak cohesive traction to minimise changes in the critical opening displacement. Non-dimensional fracture toughness, G_b/G_c, was defined as the ratio of apparent strain energy release rate to cohesive energy and non-dimensional notch radius, ρ/c, was defined as the ratio of notch radius to a critical length parameter. The critical length parameter, c, is defined as,

$$c = \frac{E' G_c}{2\pi\sigma_c^2} \tag{1}$$

Fig. 2 shows the results obtained for different peak cohesive tractions while maintaining a constant cohesive energy of 350 J/m^2. In the region $0 < \rho/c < 10$, in each case the gradient increases with ρ/c until a constant value is reached. However, for different values of peak cohesive traction, the final gradients diverge significantly. This shows there is clearly a dependence on the peak cohesive traction value used in the bilinear traction-separation law when computing the apparent fracture toughness using fracture loads. This was repeated for a range of different cohesive energies and the same divergence in final gradients was observed.

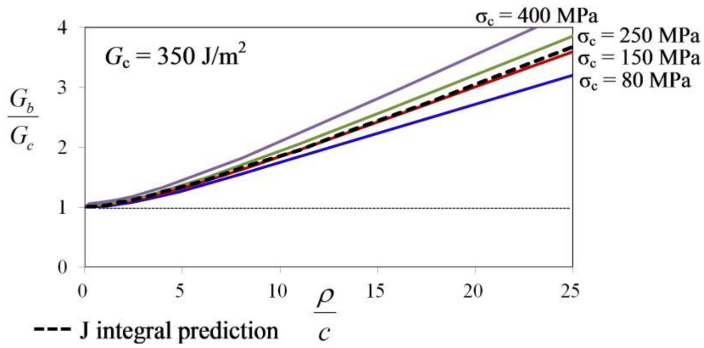

Fig. 2. Non-dimensional fracture toughness predicted using the CZM with constant cohesive energy and different peak cohesive tractions.

A similar study was conducted to investigate the effect of the cohesive energy on the non-dimensional fracture toughness by varying the cohesive energy and keeping a constant peak cohesive traction and constant initial stiffness. This is shown in Fig. 3 for a peak cohesive traction of 100 MPa. The computed results show an increase in non-dimensional fracture toughness with non-dimensional notch radius, as would be expected due to the reduced stress concentration effect of larger notch radii. In addition, as $\rho/c \to 0$, $G_b/G_c \to 1$ with an asymptote at $\rho/c = 0$. It appears that with all other parameters kept constant, the cohesive energy parameter in the traction-separation law has no significant effect on the relationship between G_b/G_c and ρ/c. Moreover, this same procedure was applied to a range of peak cohesive tractions which showed no significant dependence with cohesive energy as in the case of $\sigma_c = 100$ MPa.

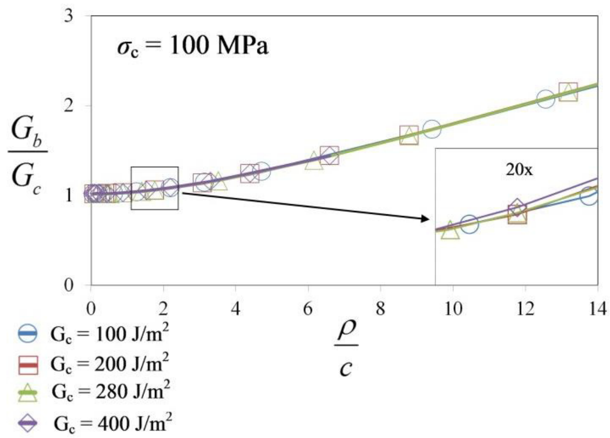

Fig. 3. Non-dimensional fracture toughness predicted using the CZM with constant peak cohesive traction and different cohesive energies.

2.4. The J-integral and comparison to analytical fracture criteria

The J contour integral was also calculated for all of the previously mentioned model parameters. The bulk material was modelled as linear elastic so the J-integral represents the strain energy release rate of the material and can be used as another measure of the apparent fracture toughness. Regardless of the cohesive energy or the peak cohesive traction values used in the traction-separation law, the J-integral gave the same relationship for G_b/G_c with ρ/c. This is shown in Fig. 2 by the dashed line. The final gradient for large ρ/c from using the J-integral was found to correspond to a peak cohesive traction of between 150 MPa and 250 MPa when using the fracture load to calculate the apparent fracture toughness. The dependence of non-dimensional fracture toughness with the peak cohesive traction shown in Fig. 2 is likely due to the cohesive zone which adds discontinuous properties to the beam.

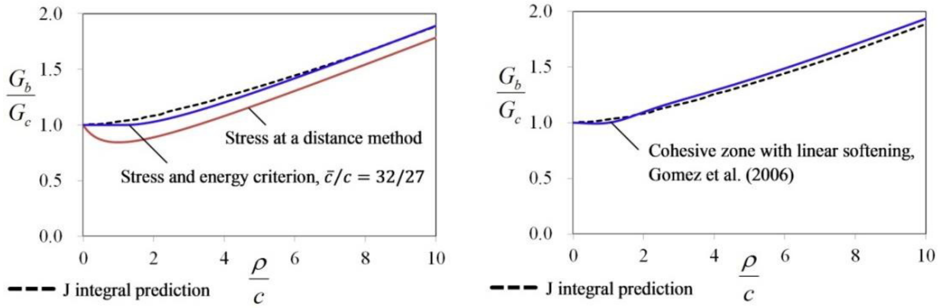

Fig. 4. Non-dimensional fracture toughness predicted from using the CZM with the J-integral compared to (a) the stress at a distance method and the stress and energy criterion, and (b) a linear softening CZM in Gomez et al. (2006).

The J-integral results are compared to the stress at a distance method, the stress and energy criterion and a linear softening cohesive zone model in Gomez et al. (2006) in Fig. 4. The cohesive zone approach and the stress and energy criterion of Williams and Patel are similar because they both fundamentally require a critical stress and a critical energy for fracture to initiate. These are the peak cohesive traction and cohesive energy respectively in the cohesive zone. Fig. 4 (a) shows close agreement between the cohesive zone model and the stress and energy criterion upper bound. However, the stress at a distance method predicts a non-dimensional fracture toughness of below unity in the range $0 < \rho/c < 1+\sqrt{5}$. Fig. 4 (b) compares the cohesive zone model using the J integral described in this paper and the cohesive zone model of Gomez et al. which is based on the use of fracture loads. Both are in close agreement in predicting the effect of notch sharpness on fracture toughness.

Finally, the convergence of non-dimensional fracture toughness using the J integral towards unity with decreasing notch radius observed in the CZM can also be observed experimentally. However, this convergence cannot be validated for very small notch radii, of the order of microns, as the notches are extremely difficult to manufacture and can introduce damage ahead of the crack tip. The CZM model described in this paper predicts an asymptotic relationship for the non-dimensional fracture toughness as a function of non-dimensional notch radius as the notch radius tends to zero.

3. Conclusion

The cohesive zone approach has been used to model fracture from blunt notches which is outside the limits of LEFM. It has been shown that the use of the fracture loads to calculate the fracture toughness has dependence on the peak cohesive traction in the traction-separation law. A better approach is to use the J-integral which gives a prediction close to the stress and energy fracture criterion.

Acknowledgements

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